

Modern Physics Letters A
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TESTS OF QUARK-HADRON DUALITY IN TAU-DECAYS

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Received (Day Month Year)

Revised (Day Month Year)

An exhaustive number of QCD finite energy sum rules for τ -decay together with the latest updated ALEPH data is used to test the assumption of global duality. Typical checks are the absence of the dimension $d = 2$ condensate, the equality of the gluon condensate extracted from vector or axial vector spectral functions, the Weinberg sum rules, the chiral condensates of dimensions $d = 6$ and $d = 8$, as well as the extraction of some low-energy parameters of chiral perturbation theory. Suitable pinched linear integration kernels are introduced in the sum rules in order to suppress potential quark-hadron duality violations and experimental errors. We find no compelling indications of duality violations in hadronic τ -decay in the kinematic region above $s \simeq 2.2 \text{ GeV}^2$ for these kernels.

Keywords: QCD; Sum Rules; Tau Decays.

PACS Nos.: 12.38.Lg, 11.55.Hx, 13.35.Dx

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1. Introduction

Tau decay constitutes the ideal laboratory to test subtle effects of QCD in the intermediate energy region which is still accessible to the perturbation series and the operator product expansion (OPE). To present the case, we collect here the results that may be obtained with minimal assumptions. The only information we use is the one that has been calculated explicitly, i.e. the known coefficients of a perturbative series and the Wilson coefficients of the OPE. The hypothesis that this approach works is known to be violated in practice, for instance, by the factorial growth of the coefficients and the presence of duality violations (DV). We assume that the asymptotic behavior sets in at fairly high perturbative orders so that it does not affect phenomenological applications considered here. As for possible DV, we minimize their impact by restricting our analysis to pinched finite energy sum rules (FESR) involving spectral function moments of low mass dimensions. Pinching serves two purposes, it reduces the contribution near the positive real axis in the contour integral of the QCD correlator and it reduces the experimental errors of the τ spectral functions. Our approach is orthogonal to a number of recent papers^{3,4} where an ansatz for DV is made based on large- N_c QCD and Regge models with the aim to identify and quantify DV. For this purpose, higher dimensional spectral moments must be considered. We want to demonstrate that for the low dimensional moments and duality radii larger than about 2.2 GeV^2 , there is little or no evidence for the existence of DV given the experimental errors of the ALEPH data⁵. To reach such a conclusion it is mandatory to consider all sum rules where the answer is known beforehand. These include the Weinberg and related sum rules for the chiral correlator and sum rules for the separate vector (V) and axial vector (A) correlator. For example, by incorporating the fact that there is no dimension two operator in QCD we can set up simple pinched FESR to demonstrate that the gluon condensate is equal and positive for the V and A correlators. The following analysis is a summary of two of our recent papers^{1,2}.

2. QCD finite energy sum rules

The strangeness conserving hadronic spectral functions in τ -decay are related to the (charged) V and A current correlators

$$\begin{aligned}\Pi_{\mu\nu}^{VV}(q^2) &= i \int d^4x e^{iqx} \langle 0 | T(V_\mu(x) V_\nu^\dagger(0)) | 0 \rangle \\ &= (-g_{\mu\nu} q^2 + q_\mu q_\nu) \Pi_V(q^2),\end{aligned}\tag{1}$$

$$\begin{aligned}\Pi_{\mu\nu}^{AA}(q^2) &= i \int d^4x e^{iqx} \langle 0 | T(A_\mu(x) A_\nu^\dagger(0)) | 0 \rangle \\ &= (-g_{\mu\nu} q^2 + q_\mu q_\nu) \Pi_A(q^2) - q_\mu q_\nu \Pi_0(q^2),\end{aligned}\tag{2}$$

where $V_\mu(x) = \bar{u}(x) \gamma_\mu d(x)$, $A_\mu(x) = \bar{u}(x) \gamma_\mu \gamma_5 d(x)$ with $u(x)$ and $d(x)$ the quark fields. Our starting point is perturbative QCD and the operator product expansion

(OPE). We write

$$\Pi^{QCD}(s) = \Pi^{PERT}(s) + \Pi^{OPE}(s) \quad (3)$$

where $\Pi^{PERT}(s)$ is the perturbation series of massless QCD known up to 5 loops¹⁴ and

$$4\pi^2 \Pi^{OPE}(Q^2) \equiv \sum_{N=2}^{\infty} \frac{1}{Q^{2N}} C_{2N}(Q^2, \mu^2) \langle 0 | \mathcal{O}_{2N}(\mu^2) | 0 \rangle, \quad (4)$$

and $Q^2 \equiv -q^2$, and the sum is over scalar gauge invariant operators of dimension 2, 4, 6, ... The parameter μ is a renormalization scale separating long from short distance physics. Short distances are absorbed in the Wilson coefficients $C_{2N}(Q^2, \mu^2)$ and long distances in the vacuum condensates $\langle \mathcal{O}_{2N}(\mu^2) \rangle$. Quark mass effects are completely negligible for the $u - d$ quark sector considered here. The relevant scale of τ -decay is m_τ which is considered to be much larger than the scale Λ_{QCD} associated with non-perturbative effects beyond the OPE.

Our normalization of the correlators is such that

$$\frac{1}{\pi} \text{Im} \Pi^{PERT}(s) = \frac{1}{4\pi^2} \left(1 + \frac{\alpha_s(\mu^2)}{\pi} + \dots \right).$$

Note that the term with $N = 1$ is absent from the OPE as, in massless QCD, there is no gauge invariant operator of dimension $d = 2$. The question of the absence of such a condensate will, however, be checked in our analysis. At dimension $d = 4$ the contribution from the gluon condensate should be equal for V and A correlators. The correlators are analytic in the complex s ($= q^2$) plane with a right-hand cut

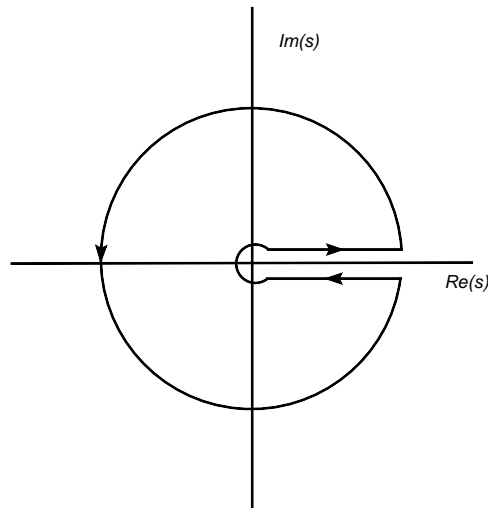


Fig. 1. Integration contour of the FESR.

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starting at the relevant thresholds. Integrating $\Pi(s)$ over the contour of Fig. 1 and applying Cauchy's theorem one obtains the following finite energy sum rule (FESR)

$$\int_0^{s_0} ds P(s) \rho_{V,A}(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} ds P(s) \Pi_{V,A}^{QCD}(s) + \text{Res}[P(s) \Pi_{V,A}(s)] , \quad (5)$$

where $\rho_{V,A}(s)$ are the spectral functions measured in τ -decay and $P(s)$ is taken here to be a power series

$$P(s) = \sum a_n s^n, \quad n = \dots, -2, -1, 0, 1, 2, \dots$$

In the convention of ALEPH⁵

$$\rho(s) = \frac{1}{2\pi^2} [v(s), a(s)]_{\text{ALEPH}} . \quad (6)$$

To the axial spectral function the pion pole has to be added $\rho_A^{\text{pion}}(s) = 2f_\pi^2 \delta(s - m_\pi^2)$.

We absorb the Wilson coefficients (ignoring radiative corrections) into the operators and write the OPE as

$$\Pi^{\text{OPE}}(Q^2) = 4\pi^2 \sum_{N=0}^{\infty} \frac{1}{Q^{2N}} \mathcal{O}_{2N} . \quad (7)$$

For $P(s)$ a polynomial, the FESR Eq. (5) can be written as

$$(-)^{N+1} \mathcal{O}_{2N+2}^{V,A} = 4\pi^2 \int_0^{s_0} ds s^N \left[\frac{\rho_V(s)}{\rho_A(s)} \right] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 2f_\pi^2 - s_0^{N+1} M_{2N+2}(s_0) , \quad (8)$$

with the PQCD moments $M_{2N+2}(s_0)$ defined as

$$M_N(s_0) \equiv \int_0^{s_0} \frac{ds}{s_0} \left[\frac{s}{s_0} \right]^N 4\pi^2 \frac{1}{\pi} \text{Im} \Pi^{\text{PQCD}}(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s_0} \left[\frac{s}{s_0} \right]^N 4\pi^2 \Pi^{\text{PQCD}}(s)$$

for both V and A . These moments are dimensionless and normalized according to

$$M_N = \frac{1}{N+1} \quad \text{for} \quad \alpha_s = 0 . \quad (9)$$

As was proposed some time ago⁶, it is advantageous to consider pinched sum rules where the weight vanishes at s_0 , the end of the integration range,

$$(-)^N \mathcal{O}_{2N+2}^{V,A} = -4\pi^2 s_0^N \int_0^{s_0} ds \left[1 - \left(\frac{s}{s_0} \right)^N \right] \rho_{V,A}(s) - s_0^{N+1} [M_0(s_0) - M_{2N+2}(s_0)] .$$

These sum rules offer two advantages. First they reduce significantly the effect of experimental errors which increase with s_0 . Second they reduce the contribution of the contour integration region near the cut where DV would be most significant.

3. Chiral sum rules

The simplest object to study is the chiral correlator $\Pi_{V-A}(s) \equiv \Pi_V(s) - \Pi_A(s)$ because it vanishes identically in the chiral limit ($m_q = 0$), to all orders in PQCD. OPE contributions start with the dimension six quark condensate,

$$\Pi(Q^2)|_{V-A}^{\text{OPE}} = -\frac{32\pi}{9} \frac{\alpha_s \langle \bar{q}q \rangle^2}{Q^6} \left\{ 1 + \frac{\alpha_s(Q^2)}{4\pi} \left[\frac{247}{12} + \ln \left(\frac{\mu^2}{Q^2} \right) \right] \right\} + \mathcal{O}(1/Q^8), \quad (10)$$

The α_s corrections were calculated⁷ in the anti-commuting γ_5 scheme and assuming vacuum saturation of the four-quark condensate.

3.1. Weinberg-type sum rules

We begin the analysis with the second Weinberg sum rule (WSR)⁸,

$$\int_0^{s_0} ds \, s [\rho_V(s) - \rho_A(s)] = 0, \quad (11)$$

Using recent ALEPH data⁵ the result is presented in Fig. 2. The sum rule is not saturated, except possibly near the endpoint of the spectrum. Due to the large experimental errors, it is not clear to what extent saturation occurs there. The same holds for the first Weinberg sum rule.

$$W_1(s_0) : \int_{s_{thr}}^{s_0} ds [\rho_V(s) - \rho_A(s)] = 2f_\pi^2. \quad (12)$$

As mentioned above, the saturation of the various sum rules can be considerably improved by introducing an integration kernel that vanishes at the upper limit of integration ($s = s_0$). Combining the first and second Weinberg sum rules we obtain the *pinched first Weinberg sum rule*,

$$W_1(s_0) : \int_{s_{thr}}^{s_0} ds \left(1 - \frac{s}{s_0} \right) [\rho_V(s) - \rho_A(s)] = 2f_\pi^2. \quad (13)$$

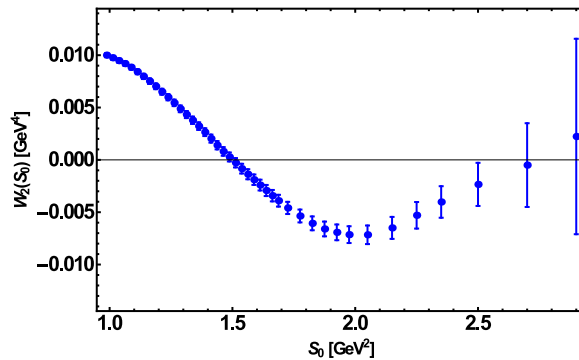


Fig. 2. The second WSR, the spectral integral of Eq. (11) should be zero.

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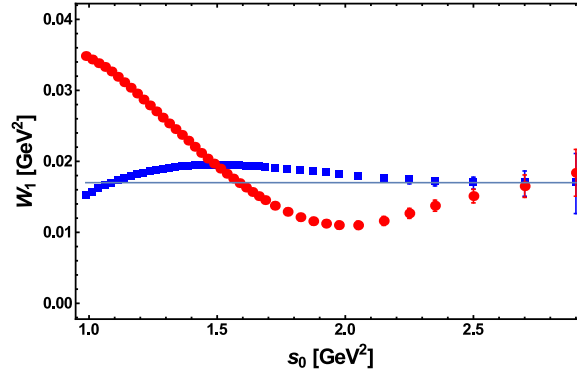


Fig. 3. First WSR (dots), Eq. (12) and the pinched WSR sum rule (squares), Eq. (13). The straight line is $2f_\pi^2$.

It can be seen from Fig. 3 that with pinching the sum rule is saturated at smaller s_0 , i.e. beginning at $s_0 = 2.2 \text{ GeV}^2$. In addition the errors from experiment are substantially reduced. From Fig. 3 we would extract

$$f_\pi^2 = 0.008 \pm 0.004 \text{ GeV}^2 ,$$

for curve (a), and

$$f_\pi^2 = 0.0084 \pm 0.0004 \text{ GeV}^2 ,$$

for curve (b), to be compared with the experimental value $f_\pi^2|_{EXP} = 0.00854 \pm 0.00005 \text{ GeV}^2$. Curve (a) demonstrates incidentally that it may be dangerous to pick up only a small stability region to obtain a prediction (here one could choose the region around 2 GeV^2).

3.2. The DGLMY sum rule

With the help of the second Weinberg sum rule, the Das-Guralnik-Low-Mathur-Young (DGLMY)¹² sum rule can also be written in a pinched form, albeit with logarithmic pinching,

$$W_3(s) \equiv \int_0^{s_0 \rightarrow \infty} ds s \ln \frac{s}{s_0} [\rho_V(s) - \rho_A(s)] = -\frac{4\pi f_\pi^2}{3\alpha} (m_{\pi^\pm}^2 - m_{\pi^0}^2) .$$

It is seen from Fig. 4 that the sum rule is saturated to the extent expected from the specific kernel. The central sum rule result for the mass difference is a few percent larger than the experimental value.

3.3. The DMO sum rule

For the negative power kernels the FESR are satisfied better because the influence of experimental errors is reduced. The second Weinberg sum rule is again exploited

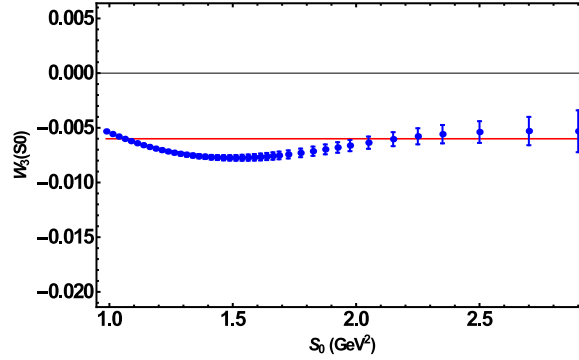


Fig. 4. The DGLY sum rule.

to generate pinching. The corresponding pinched Das-Mathur-Okubo (DMO)⁹ sum rule reads

$$\bar{\Pi}(0) = 4 \frac{f_\pi^2}{s_0} + \int_0^{s_0} \frac{ds}{s} \left(1 - \frac{s}{s_0}\right)^2 [\rho_V(s) - \rho_A(s)] . \quad (14)$$

where $\bar{\Pi}(0) = \Pi(0)$ minus the pion pole. In lowest order CHPT⁹

$$\begin{aligned} \bar{\Pi}(0) &= -8\bar{L}_{10} \\ &= 2 \left[\frac{1}{3} f_\pi^2 \langle r_\pi^2 \rangle - F_A \right] = 0.052 \pm 0.002 , \end{aligned}$$

or $\bar{L}_{10} = -(6.33 \times 10^{-3} \pm 0.06)$. Here $\langle r_\pi^2 \rangle = 0.439 \pm 0.008 \text{ fm}^2$ is the electromagnetic radius of the pion¹⁰, and $F_A = 0.0119 \pm 0.0001$ the radiative pion decay constant¹¹. Our result is plotted in Fig. 5. Numerically we get

$$\bar{L}_{10} = -(6.5 \pm 0.1) \times 10^{-3} . \quad (15)$$

It is seen that pinching is hardly necessary for this kernel. There is full agreement with alternative calculations^{3,4} and also with lattice QCD calculations¹³ within their larger uncertainties.

The FESR for the first derivative of the chiral correlator $\Pi'(0)$ is related to the $\mathcal{O}(p^6)$ counter terms. The pinched sum rule reads

$$\bar{\mathcal{C}}_{87} = \Pi'(0) + \frac{2f_\pi^2}{m_\pi^4} = \int_0^{s_0} \left(1 - \frac{s^3}{s_0^3}\right) \frac{ds}{s^2} (\rho_V(s) - \rho_A(s)) . \quad (16)$$

This value agrees within errors with the value obtained in Ref.⁴. We conclude that for negative moments the experimental errors at the large energy region of the spectral integral become increasingly less important. Pinching brings no improvement to these FESR. The agreement with alternative results is excellent.

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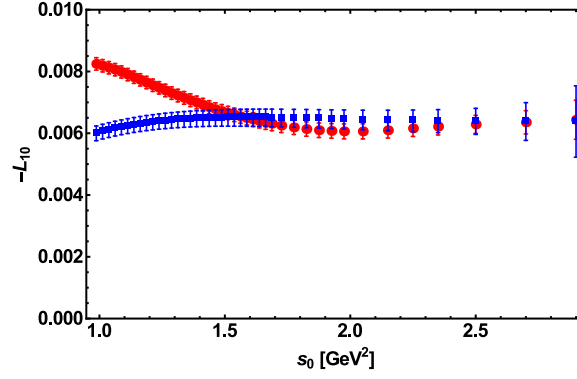


Fig. 5. The CHPT constant $-\bar{L}_{10}$ obtained from the pinched chiral sum rule for $\bar{\Pi}(0)$ Eq. (15). Red dots correspond to no pinching, blue squares to pinching.

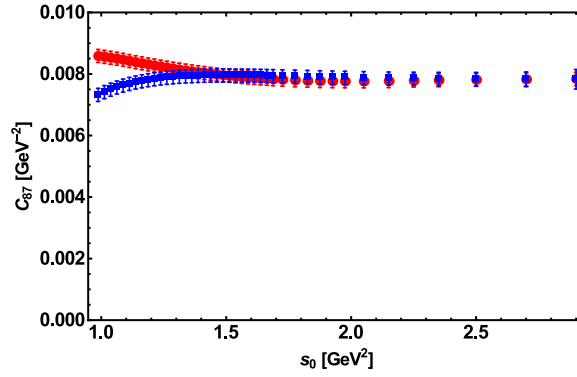


Fig. 6. The $O(p^6)$ counterterm \bar{C}_{87} of CHPT according to the sum rule Eq. (16), squares with pinching, dots without pinching.

3.4. Chiral condensates

Due to the absence of PQCD contributions, it becomes feasible to extract chiral condensates with the help of pinched FESR to reasonable accuracy. The sum rule for the dimension 6 chiral condensate reads

$$\langle \mathcal{O}_6 \rangle = -2 f_\pi^2 s_0^2 + s_0^2 \int_0^{s_0} ds \left(1 - \frac{s}{s_0} \right)^2 [\rho_V(s) - \rho_A(s)] . \quad (17)$$

The results are plotted in Fig. 7. From this FESR we extract

$$\langle \mathcal{O}_6 \rangle = -(5.0 \pm 0.7) \times 10^{-3} \text{ GeV}^6 . \quad (18)$$

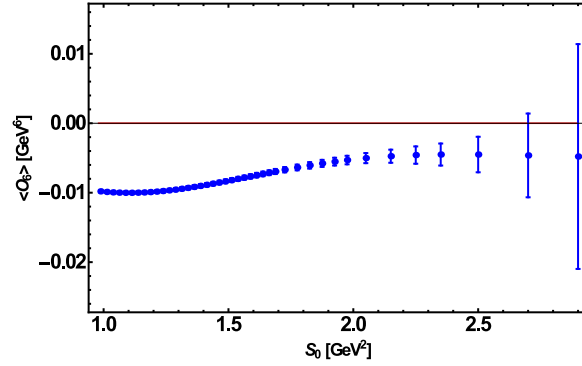


Fig. 7. The chiral condensate of dimension $d = 6$ from the pinched chiral sum rule Eq. (17).

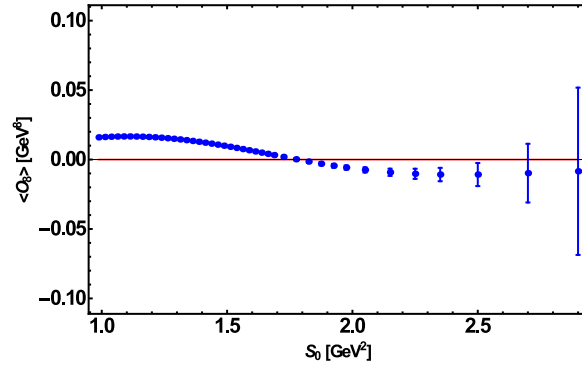


Fig. 8. The chiral condensate of dimension $d = 8$ from the pinched chiral sum rule Eq. (19).

By the same token, we can write a pinched FESR for the dimension 8 chiral condensate,

$$\langle \mathcal{O}_8 \rangle = 16 f_\pi^2 s_0^3 - 3 s_0^4 \bar{\Pi}(0) + s_0^3 \int_0^{s_0} \frac{ds}{s} \left(1 - \frac{s}{s_0} \right)^3 (s + 3 s_0) [\rho_V(s) - \rho_A(s)] . \quad (19)$$

The results are plotted in Fig. 8. As expected the dimension 8 condensate can be extracted from the sum rule only with larger error. We find

$$\langle \mathcal{O}_8 \rangle = -(9.0 \pm 5.0) \times 10^{-3} \text{ GeV}^8 .$$

4. Non-Chiral Sum Rules

When one considers QCD sum rules for the vector and the axial vector separately, the perturbative correlators contribute. To compare with QCD orthodoxy, we should

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verify that there is no dimension 2 condensate. This can be done with the sum rule

$$\frac{1}{4\pi^2} \langle \mathcal{O}_2 \rangle = - \int_{s_{thr}}^{s_0} ds \left[\frac{\rho_V(s)}{\rho_A(s)} \right] - \begin{bmatrix} 0 \\ 1 \end{bmatrix} 2f_\pi^2 + s_0 \frac{1}{4\pi^2} M_0(s_0) . \quad (20)$$

We evaluate the moments with contour improved QCD¹⁵. As an input we use

$$\alpha_s(m_\tau^2) = 0.341 \pm 0.013$$

obtained in a recent analysis¹⁷ based on similar assumptions as the ones made here. The results for the $V + A$ correlator for the two extreme values of $\alpha_s(m_\tau^2)$ are plotted in Fig. 9. It is seen that within the experimental errors there is no evidence for the existence of a non-vanishing dimension 2 operator. We use this fact to construct pinched sum rules.

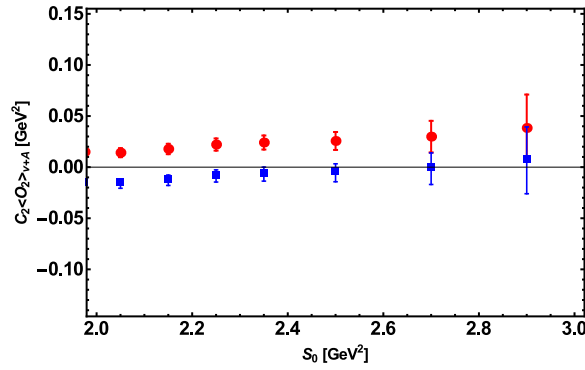


Fig. 9. $\langle \mathcal{O}_2 \rangle$ from the sum rule Eq. (20) for $\alpha_s = 0.354$ (dots) and $\alpha_s = 0.328$ (squares).

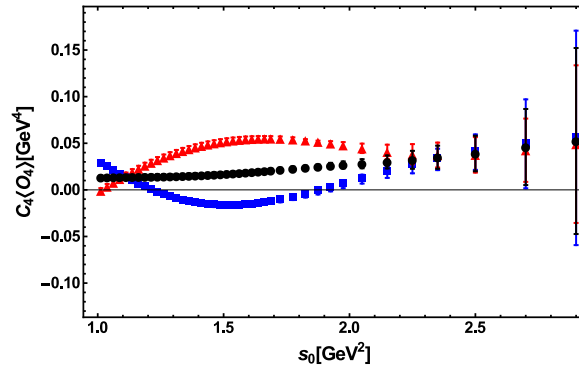


Fig. 10. $\langle \mathcal{O}_4 \rangle$ from the sum rule Eq. (21) for the V (triangles), A (squares) and $\frac{1}{2}(V + A)$ (dots) correlator.

As a first application, we consider the dimension 4 condensate for the vector and the axial vector channel

$$\begin{aligned} \frac{1}{4\pi^2} \langle \mathcal{O}_4^{V,A} \rangle &= s_0 \int_{s_{thr}}^{s_0} ds \left(1 - \frac{s}{s_0} \right) \left[\rho_V(s) \right] - \left[\begin{matrix} 0 \\ 1 \end{matrix} \right] 2f_\pi^2 \\ &\quad - s_0^2 \frac{1}{4\pi^2} [M_0(s_0) - M_1(s_0)] \end{aligned} \quad (21)$$

In the chiral limit, i.e. for massless quarks, the gluon condensates for the V and A correlators should be equal. Figure 10 shows beautifully how this is realized by the pinched sum rules. Note that for lower s_0 , the duality violations tend to cancel between V and A . From the figure we extract a value for the gluon condensate

$$\langle \mathcal{O}_4^{V,A} \rangle = \frac{\pi^2}{3} \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle = (0.017 \pm 0.012) \text{ GeV}^2$$

which is positive and equal for V and A .

Given the condensates from independent sources, we can invert the sum rule to calculate the pion decay constant

$$\begin{aligned} 2f_\pi^2 &= - \int_{s_{thr}}^{s_0} ds \left(1 - \frac{s}{s_0} \right) \rho_A(s) \\ &\quad - \frac{1}{2\pi i} \oint_{|s|=s_0} ds \left(1 - \frac{s}{s_0} \right) 4\pi^2 \Pi_A^{QCD}(s) + \frac{1}{4\pi^2} [\langle \mathcal{O}_2 \rangle - \langle \mathcal{O}_4 \rangle] . \end{aligned} \quad (22)$$

For popular values of $\langle \alpha_s GG \rangle$ the contribution of the condensates is negligible. The result is plotted in Fig. 11. A more sensitive test is produced by the vector correlator,

$$F(s_0) = - \int_{s_{thr}}^{s_0} ds \left(1 - \frac{s}{s_0} \right) \rho_V(s) - \frac{1}{2\pi i} \oint_{|s|=s_0} ds \left(1 - \frac{s}{s_0} \right) 4\pi^2 \Pi_V^{QCD}(s) . \quad (23)$$

In this case the left-hand side of Eq. (22), which we define to be $F(s_0)$, vanishes. We plot the results in Fig. 11 and Fig. 12 both for our duality approach and for a popular model of DV¹⁶ (see Ref. ² for details). As expected the results of the latter are better for small s_0 . For the contour radius larger than about 2.2 GeV², however, both approaches are equivalent.

5. Conclusions

Duality with pinching works well in τ -decay for $s_0 \gtrsim 2.3 \text{ GeV}^2$ to m_τ^2 for all observables where the answer is known. The highlights of our analysis are the early saturation of the Weinberg sum rules, the equality of the gluon condensate extracted from the V and A spectral functions, the extraction of chiral condensates and of some low-energy constants of CHPT. We use only linear kernels because of unknown higher dimensional condensates and possibly enhanced DV for higher powers. Within the experimental errors DV become unobservable for the pinched weights considered. It is essential that all QCD constraints regarding condensates

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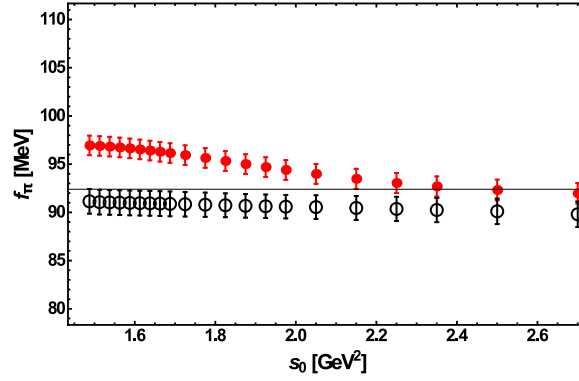


Fig. 11. The pion decay constant from the sum rule Eq. (22) assuming duality (dots) and for a model of DV (circles).

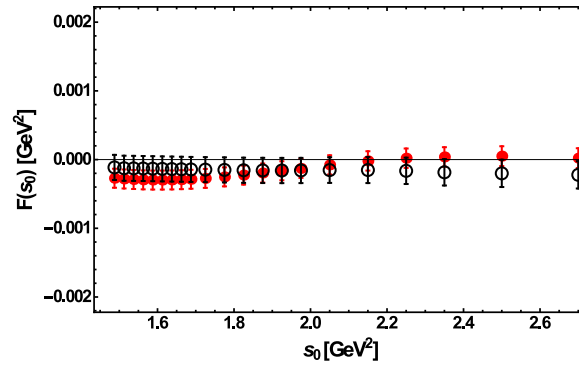


Fig. 12. $F(s_0)$ from the sum rule Eq. (23) assuming duality (squares) and for a model of DV (dots).

are incorporated. It remains to be seen if sum rules with higher powers of the pinch kernel can be satisfied in the straightforward FESR approach. The answer to this question would require a systematic analysis allowing for higher dimensional vacuum condensates and their radiative corrections.

Acknowledgements

We thank D. Boito, M. Golterman, K. Maltman and S. Peris for discussions about their work. One of us (K.S.) thanks K. Maltman for a correspondence. We acknowledge financial support by the Deutsche Forschungsgemeinschaft, the Mainz Institute for Theoretical Physics (MITP), and the National Research Foundation (South Africa).

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